International Journal of Engineering and Applied Sciences (IJEAS)
ISSN: 2394-3661, Volume-2, Issue-11, November 2015

Modified Iterated Square-root Cubature Kalman Filter for Non-cooperative Space Target Tracking

Chaochen Wang, Panlong Wu, Yuhao Deng

***Abstract*—Passive tracking techniques for non-cooperative space target have great significance in space surveillance systems. In this paper, we proposed a new filtering algorithm for passive tracking problem called iterated square-root cubature Kalman filter (ISCKF). By introducing a Newton-Gauss iterative method into the square-root cubature Kalman filter (SCKF), the proposed filtering algorithm has a better filtering performance in accuracy and stability. The simulation results demonstrate that the ISCKF outperforms the conventional filters when using bearings-only measurements.**

***Index Terms*—Non-cooperative space target, passive tracking, bearings-only, ISCKF.**

I. Introduction

The surveillance of space objects is the important approach to obtain national space stratagem information [1-2]. Compared with the cooperative tracking mode, the satellite-to-satellite passive tracking system can obtain angles and frequencies by means of optical or radioed measurements [3-4]. The space-based bearings-only tracking and orbit determination technology for non-cooperative space target (NCST) is a critical technology for realizing and establishing our space-based surveillance system [5]. It can be used in identification of newly launched satellites, collision avoidance, orbit maneuver [6-7], etc. For example Low earth orbit (LEO) satellites usually need signals relayed to transmit signals to the ground station. If a high earth orbit (HEO) satellite is used to passively receive these signals, such as navigation and communication. The satellite tracking system can be realized through analysis and can estimate the parameters of these signals using filtering algorithms.

Considering the nonlinear state and measurement system, a nonlinear filter is required for state estimation. Several nonlinear filters have been proposed to solve the nonlinear filtering problem, such as an extended Kalman filter (EKF), an unscented Kalman filter (UKF), a particle filter (PF) [8], etc. However, these filters have various defects, for example EKF performs badly in strong nonlinear filtering problem; PF has an enormous amount of computation in high dimension problem; UKF might have a non-positive covariance when the dimension of the system is more than three and it will lead to the unstable filtering performance of UKF. The newly proposed cubature Kalman filter (CKF) [9] applies a third-degree spherical-radial cubature rule for numerically computing Gaussian weighted integrals. The spherical-radial cubature rule leads to an even number of equally-weighted cubature points. These cubature points are distributed uniformly on a sphere centered at the origin. In this dissertation, a modified CKF called iterated square-root cubature Kalman filter (ISCKF) using bearings-only measurements is proposed for non-cooperative space target tracking problem. The iterative algorithm [10-11] is introduced into CKF to increase the filtering accuracy and numerical stability.

The rest of the paper is organized as follows. The tracking model for the satellite passive tracking is formulated in Section 2, including the dynamic model of the target satellite and bearings-only measurement model. Section 3 presents proposed ISCKF algorithm in details. In Section 4, the Monte Carlo simulation results of the proposed ISCKF are shown in a designed scenario. The conclusion is given in Section 5.

II. Tracking Model

In this dissertation, a scenario is designed to build the tracking model. The geometrical relationship of the earth, the observation satellite and the target satellite are shown in Fig. 1, where O represents the observation satellite and T represents the target satellite, respectively. According to Fig. 1, the dynamic model and measurement model are given as blow.

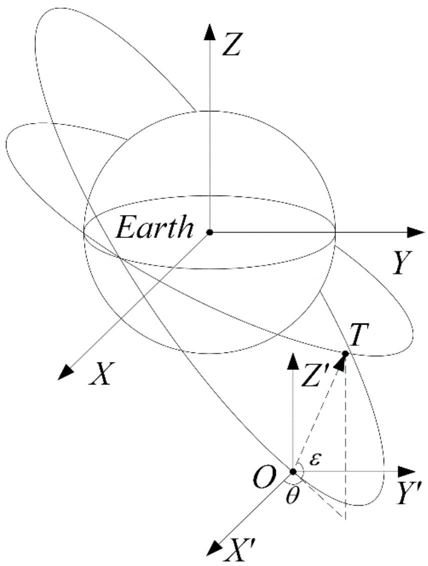


Fig. 1. The geometrical relationship of the earth, the observation platform and the tracking target

A. *Dynamic Model*

Considering the J2 perturbation effect of the earth gravity, the dynamic model of the target satellite can be built in earth

This work is partially supported by National Nature Science Foundation of China (no. 61301217, 61473153), Research Fund for the Doctoral Program Higher Education of China (no. 20123219120043), Natural Science Foundation of Jiangsu Province (BK20131352) and A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions(PAPD).

Chaochen Wang, the School of Automation, Nanjing University of Science and Technology, Nanjing 210094.

Panlong Wu, the School of Automation, Nanjing University of Science and Technology, Nanjing 210094.

Yuhao Deng, the School of Automation, Nanjing University of Science and Technology, Nanjing 210094.

J2000 coordinate system as

$$\begin{cases} \dot{x} = v_x \\ \dot{y} = v_y \\ \dot{z} = v_z \\ \dot{v}_x = -\dfrac{\mu_e x}{|r|^3}\left[1 - J_2\left(\dfrac{R_e}{|r|}\right)^2\left(7.5\dfrac{z^2}{|r|^2} - 1.5\right)\right] \\ \dot{v}_y = -\dfrac{\mu_e y}{|r|^3}\left[1 - J_2\left(\dfrac{R_e}{|r|}\right)^2\left(7.5\dfrac{z^2}{|r|^2} - 1.5\right)\right] \\ \dot{v}_z = -\dfrac{\mu_e z}{|r|^3}\left[1 - J_2\left(\dfrac{R_e}{|r|}\right)^2\left(7.5\dfrac{z^2}{|r|^2} - 4.5\right)\right] \end{cases} \tag{1}$$

where $r = (x, y, z)^T$ and $v = (v_x, v_y, v_z)^T$ are the position and velocity of the target satellite with respect to the earth center. μ_e is the earth gravity constant. J_2 is the coefficient of zonal harmonic terms. R_e is the radius of the earth. Equation (1) can be described as

$$\dot{X} = f\left(X(t)\right) + w(t) \tag{2}$$

where $X(t) = \left[x, y, z, v_x, v_y, v_z\right]^T$ is state vector of the target. $w(t) = [w_x, w_y, w_z, w_{v_x}, w_{v_y}, w_{v_z}]^T$ is the system process noise which is assumed to be a Gaussian white noise.

B. *Measurement Model*

At time *k*, the azimuth angle θ and pitching angle ε between the observation satellite and the target satellite can be obtained, which are defined as

$$\theta_k = \arctan\left(\frac{y_k - y_{O,k}}{x_k - x_{O,k}}\right) \tag{3}$$

$$\varepsilon_k = \arctan\left(\frac{z_k - z_{O,k}}{\sqrt{\left(x_k - x_{O,k}\right)^2 + \left(y_k - y_{O,k}\right)^2}}\right) \tag{4}$$

where $r_O = \left[x_O, y_O, z_O\right]^T$ is the position of the observation platform with respect to the earth center.

According to (3) and (4), the measurement model is expressed as

$$Z_k = \begin{bmatrix} \theta_k \\ \varepsilon_k \end{bmatrix} = h\left(X_k\right) + v_k \tag{5}$$

where $v_k = \left[v^{(1)}, v^{(2)}\right]^T$ is the measurement noise which is assumed to be a Gaussian white noise.

III. Iterated Square-root Cubature Kalman Filter

A. *Square-root Cubature Kalman Filter*

Arasaratnam and Haykin present a new nonlinear filter for high-dimensional state estimation, which we have named the cubature Kalman filter (CKF) in [9]. They derived a third-degree spherical-radial cubature rule that provides a set of cubature points scaling linearly with the state-vector dimension. The CKF may therefore provide a systematic solution for high-dimensional nonlinear filtering problems. At the same time, a square-root version of CKF called square-root cubature Kalman filter (SCKF) is also given in [9]. Compared with some conventional filters, SCKF has higher filtering accuracy and numerical stability. The algorithm is given below.

Time Update

1) Evaluate the cubature points ($i = 1, 2, \dots, m$)

$$X_{i,k-1|k-1} = S_{k-1|k-1}\xi_i + \hat{x}_{k-1|k-1} \tag{6}$$

where $m = 2n_x$ and when $k = 1$,

$$S_{0|0} = sqrt\left(P_0\right) \tag{7}$$

2) Evaluate the propagated cubature points ($i = 1, 2, \dots, m$)

$$X^*_{i,k|k-1} = f\left(X_{i,k-1|k-1}\right) \tag{8}$$

where $f\left(X\right)$ is given in (2).

3) Estimate the predicted state

$$\hat{x}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} X^*_{i,k|k-1} \tag{9}$$

4) Estimate the square-root factor of the predicted error covariance

$$S_{k|k-1} = Tria\left(\left[x^*_{k|k-1}, S_{Q,k-1}\right]\right) \tag{10}$$

where $S_{Q,k-1}$ denotes a square-root factor of Q_{k-1} such that $Q_{k-1} = S_{Q,k-1}S^T_{Q,k-1}$, and $\chi^*_{k|k-1}$ defined as

$$\chi^*_{k|k-1} = \frac{1}{\sqrt{m}}\left[X^*_{1,k|k-1} - \hat{x}_{k|k-1}, X^*_{2,k|k-1} - \hat{x}_{k|k-1}, \dots, X^*_{m,k|k-1} - \hat{x}_{k|k-1}\right] \tag{11}$$

Measurement Update

1) Evaluate the cubature points ($i = 1, 2, \dots, m$)

$$X_{i,k|k-1} = S_{k|k-1}\xi_i + \hat{x}_{k|k-1} \tag{12}$$

2) Evaluate the propagated cubature points ($i = 1, 2, \dots, m$)

$$Z_{i,k|k-1} = h\left(X_{i,k|k-1}\right) \tag{13}$$

where $h\left(X\right)$ is given in (5).

3) Estimate the predicted measurement

$$\hat{z}_{k|k-1} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1} \tag{14}$$

4) Estimate the square-root of the innovation covariance matrix

$$S_{zz,k|k-1} = Tria\left(\left[Z_{k|k-1}, S_{R,k}\right]\right) \tag{15}$$

where $S_{R,k}$ denotes a square-root factor of R_k such that $R_k = S_{R,k}S^T_{R,k}$, and $Z_{k|k-1}$ is defined as

$$Z_{k|k-1} = \frac{1}{\sqrt{m}}\left[Z_{1,k|k-1} - \hat{z}_{k|k-1}, Z_{2,k|k-1} - \hat{z}_{k|k-1}, \dots, Z_{m,k|k-1} - \hat{z}_{k|k-1}\right] \tag{16}$$

5) Estimate the cross-covariance matrix

$$P_{xz,k|k-1} = \chi_{k|k-1}Z^T_{k|k-1} \tag{17}$$

where $\chi_{k|k-1}$ is defined as

$$\chi_{k|k-1} = \frac{1}{\sqrt{m}}\left[X_{1,k|k-1} - \hat{x}_{k|k-1}, X_{1,k|k-1} - \hat{x}_{k|k-1}, \dots, X_{m,k|k-1} - \hat{x}_{k|k-1}\right] \tag{18}$$

6) Estimate the Kalman gain

$$P_{zz,k|k-1} = S_{zz,k|k-1}S^T_{zz,k|k-1} \tag{19}$$

$$W_k = P_{xz,k|k-1} \Big/ \left(P_{zz,k|k-1} + R_k\right) \tag{20}$$

7) Estimate the updated state

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k\left(z_k - \hat{z}_{k|k-1}\right) \tag{21}$$

8) Estimate the square-root factor of the corresponding error covariance

$$S_{k|k} = Tria\left(\left[\chi_{k|k-1} - W_k Z_{k|k-1}, W_k S_{R,k}\right]\right) \tag{22}$$

B. Iterated Square-root Cubature Kalman Filter

In this dissertation, a modified SCKF is proposed called iterated square-root cubature Kalman filter (ISCKF). The Newton-Gauss iterative algorithm [12-13] is introduced in SCKF to improve the filtering accuracy and stability. The ISCKF is given as blow.

Time Update is the same as SCKF.

Measurement Update

1) Evaluate the cubature points ($i = 1, 2, ..., m$)

$$X_{i,k|k-1}^{(j)} = \hat{S}_{k|k-1}^{(j)}\xi_i + \hat{x}_{k|k-1}^{(j)} \tag{23}$$

where $j = 0, 1, ..., N$ and when $j = 0$, $\hat{S}_{k|k-1}^{(0)} = S_{k|k-1}$ and $\hat{x}_{k|k-1}^{(0)} = \hat{x}_{k|k-1}$.

2) Evaluate the propagated cubature points ($i = 1, 2, ..., m$)

$$Z_{i,k|k-1}^{(j)} = h\left(X_{i,k|k-1}^{(j)}\right) \tag{24}$$

where $h(X)$ is given in (5).

3) Estimate the predicted measurement

$$\hat{z}_{k|k-1}^{(j)} = \frac{1}{m}\sum_{i=1}^{m} Z_{i,k|k-1}^{(j)} \tag{25}$$

4) Estimate the square-root of the innovation covariance matrix

$$S_{zz,k|k-1}^{(j)} = Tria\left(\left[Z_{k|k-1}^{(j)}, S_{R,k}^{(j)}\right]\right) \tag{26}$$

where $S_{R,k}^{(j)}$ denotes a square-root factor of $R_k^{(j)}$ such that $R_k^{(j)} = S_{R,k}^{(j)}\left(S_{R,k}^{(j)}\right)^T$, and $Z_{k|k-1}^{(j)}$ is defined as

$$Z_{k|k-1}^{(j)} = \frac{1}{\sqrt{m}}\left[Z_{1,k|k-1}^{(j)} - \hat{z}_{k|k-1}^{(j)}, Z_{2,k|k-1}^{(j)} - \hat{z}_{k|k-1}^{(j)}, ..., Z_{m,k|k-1}^{(j)} - \hat{z}_{k|k-1}^{(j)}\right] \tag{27}$$

5) Estimate the cross-covariance matrix

$$P_{xz,k|k-1}^{(j)} = \chi_{k|k-1}^{(j)}\left(Z_{k|k-1}^{(j)}\right)^T \tag{28}$$

where $\chi_{k|k-1}^{(j)}$ is defined as

$$\chi_{k|k-1}^{(j)} = \frac{1}{\sqrt{m}}\left[X_{1,k|k-1}^{(j)} - \hat{x}_{k|k-1}^{(j)}, X_{2,k|k-1}^{(j)} - \hat{x}_{k|k-1}^{(j)}, ..., X_{m,k|k-1}^{(j)} - \hat{x}_{k|k-1}^{(j)}\right] \tag{29}$$

6) Estimate the Kalman gain

$$P_{zz,k|k-1}^{(j)} = S_{zz,k|k-1}^{(j)}\left(S_{zz,k|k-1}^{(j)}\right)^T \tag{30}$$

$$W_k^{(j)} = P_{xz,k|k-1}^{(j)} \Big/ \left(P_{zz,k|k-1}^{(j)} + R_k^{(j)}\right) \tag{31}$$

7) Estimate the updated state

$$\hat{x}_{k|k}^{(j)} = \hat{x}_{k|k-1}^{(j)} + W_k^{(j)}\left(z_k^{(j)} - \hat{z}_{k|k-1}^{(j)}\right) \tag{32}$$

8) Estimate the square-root factor of the corresponding error covariance

$$S_{k|k}^{(j)} = Tria\left(\left[\chi_{k|k-1}^{(j)} - W_k^{(j)} Z_{k|k-1}^{(j)}, W_k^{(j)} S_{R,k}^{(j)}\right]\right) \tag{33}$$

9) Make $\hat{x}_{k|k-1}^{(j+1)} = \hat{x}_{k|k}^{(j)}$, $\hat{S}_{k|k-1}^{(j+1)} = S_{k|k}^{(j)}$ and $j = j+1$. Return to Measurement Update Step 1 and end for $j = N$.

IV. Simulations

To demonstrate the validity and reliability of the proposed algorithm, this section gives the simulation results. The normal orbit of the target satellite and the observation satellite are generated by Satellite Tool Kit (STK). The orbit elements of the two satellites are given in Table 1.

Monte Carlo simulation results of the orbit determination performance for the EKF, SCKF and ISCKF are presented, and 100 runs are performed. Some initial parameters are defined as follows: Sampling time is 1s. The covariance of state process noise Q=diag([$1^2 m$, $1^2 m$, $1^2 m$, $0.01^2 m/s$, $0.01 m/s$, $0.01^2 m/s$]). The covariance of the measurement noise R=diag([$20\mu rad$, $20\mu rad$]). The initial state error δX =[10*km*, 10*km*, 10*km*, 5*m/s*, 5*m/s*, 5*m/s*]. $\mu_e = 3.986005 \times 10^{14} m^3/s^2$, J_2=0.00108263, R_e=6371km. The iteration number N is 5.

The trajectories of the satellites are shown in Fig. 2. Figs. 3-5 show the position and velocity estimation error of EKF, SCKF and ISCKF, respectively. Table 2 shows the RMSE of the three algorithm and Table 3 shows the position and velocity estimation error when the filtering results are stable.

Table 1 The orbit elements of the target satellite and the observation satellite

Orbit element	The observation satellite	The target satellite
Semimajor Axis(km)	8000	14000
Eccentricity	0	0
Inclination (deg)	5	50
RAAN (deg)	0	0
Argument of Perige (deg)	0	0
Mean Anomaly (deg)	200	218

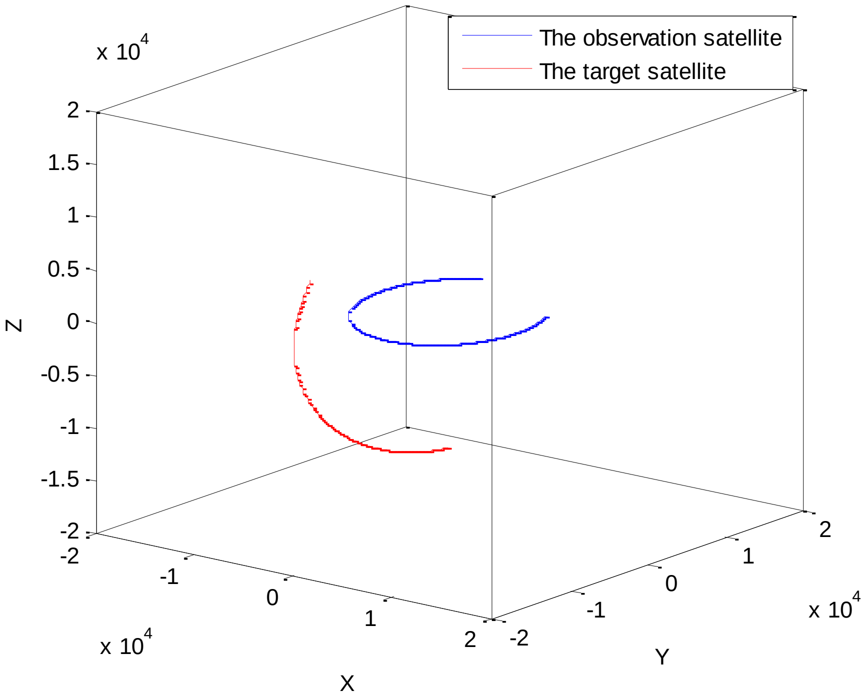


Fig. 2. The trajectories of the two satellites

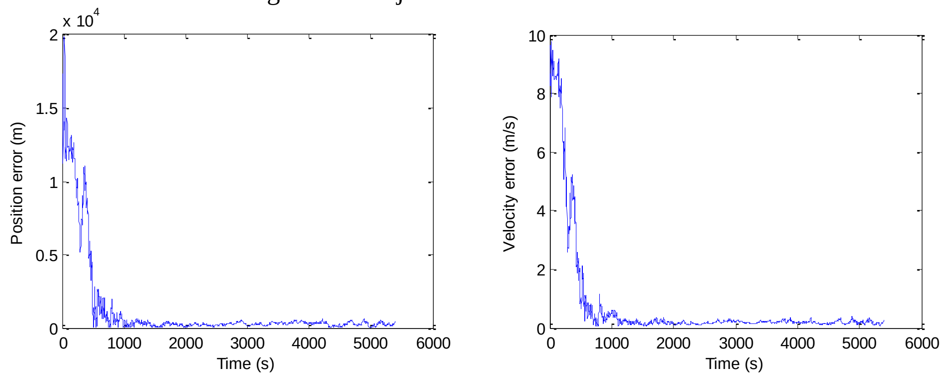


Fig. 3. Estimation error of EKF

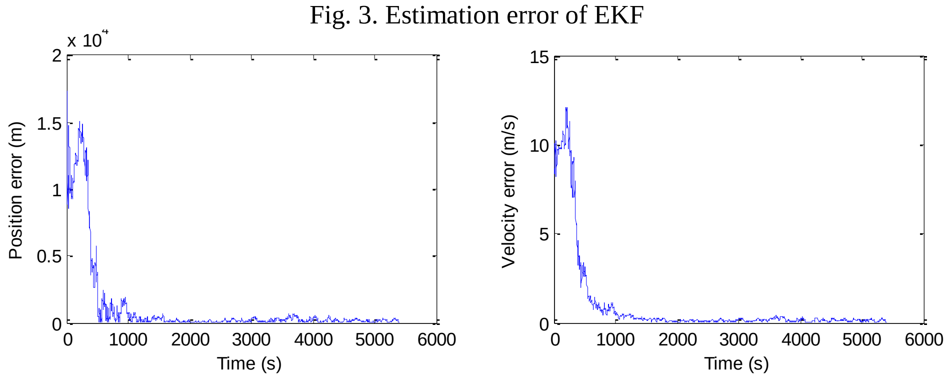


Fig. 4. Estimation error of SCKF

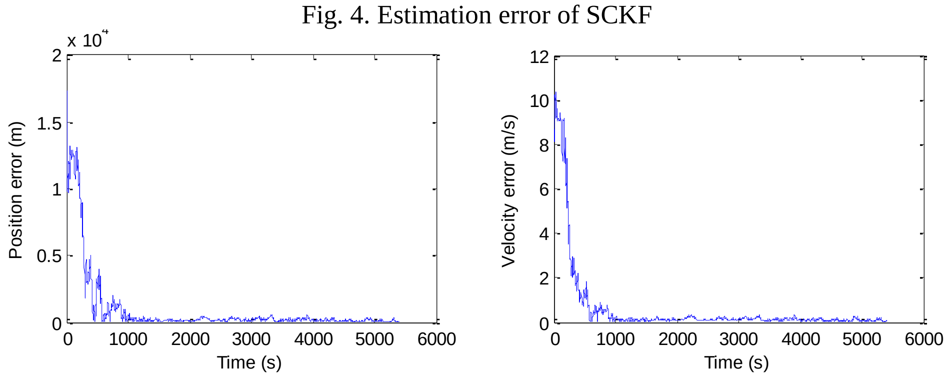


Fig. 5. Estimation error of ISCKF

Table 2 The RMSE of EKF, SCKF and ISCKF

Algorithm	Position RMSE (m)	Velocity RMSE (m/s)
EKF	1206.600	0.746
SCKF	1157.508	0.947
ISCKF	906.288	0.602

Table 3 The estimation error of EKF, SCKF and ISCKF when filtering results are stable

Algorithm	Position error (m)	Velocity error (m/s)
EKF	269.351	0.189
SCKF	166.796	0.136
ISCKF	127.082	0.102

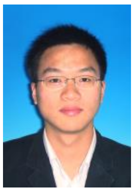
Summarizing the above simulation results, the performance of ISCKF is better than the other two filters. From the figures, the stability and accuracy of ISCKF outperforms the other two filters. From the Table 2, the position RMSE of ISCKF decreases by 24.89% and 21.70% compared with SCKF and ISCKF, respectively, and the velocity RMSE of ISCKF decreases by 19.30% and 36.43% compared with SCKF and ISCKF, respectively. From the Table 3, the stable position error of ISCKF decreases by 52.82% and 23.81% compared with SCKF and ISCKF, respectively, and the stable velocity error of ISCKF decreases by 46.03% and 25.00% compared with SCKF and ISCKF, respectively.

V. Conclusion

In this paper, we proposed a new filtering algorithm for passive tracking problem called iterated square-root cubature Kalman filter (ISCKF). The Newton-Gauss iterative method is introduced into ISCKF to improve the conventional SCKF. The simulation results show that when using bearings-only measurements in tracking system, the proposed ISCKF outperforms the EKF and SCKF by comparison. It has a better filtering performance in accuracy and stability. The proposed ISCKF is an effective algorithm for passive target satellite tracking systems.

References

[1] S.X. Zhang, Trajectory Tracking and Satellite Orbit Measurement, Beijing, China, National Defense Industry Press, 1992.
[2] Q. Li, L.H. Shi, H.X. Wang, F.C. Guo, "Observability of satellite-to-satellite three dimensional passive tracking using bearing-only measurements", Journal of Astronautics, Vol. 30, pp. 1957–1966, 2009.
[3] G. Jauffret, D. Pillon, "Observability in passive target motion analysis", IEEE Transactions on Aerospace and Electronic Systems, Vol. 32, pp. 1290–1300, 1996.
[4] Jayant S, Curt von B. Toward Operational Space-based space Surveillance. Lincoln Laboratory Journal, 2002, 13 (2):309-334
[5] Escobal P R. Methods of Orbit Determination [M]. John Wiley & Sons, Inc. , New York, 1965:187-358
[6] Becker, K. Three-Dimensional Target Motion Analysis Using Angles and Frequency Measurements [J]. IEEE Transactions on Aerospace and Electronic Systems, 2005, 41(1):284~301.
[7] Ho, K. C., Chan, Y. T. An Asymptotically Unbiased Estimator for Bearings-Only and Doppler-Bearing Target Motion Analysis[J]. IEEE Transactions on Signal Processing, 2006, 54(3):809~822.
[8] Julier, S. and Uhlmann, J. A new extension of the Kalman filter to nonlinear systems. In Proceedings of SPIE, vol. 3068, Apr. 1997, pp.182—193.
[9] I. Arasaratnam and S. Haykin, "Cubature kalman filters," IEEE Transactions on Automatic Control, vol. 54, no. 6, pp. 1254–1269, 2009.
[10] L.Q. Li, H.B. Ji, J.H. Luo, "The iterated extended Kalman particle filter", IEEE International Symposium on Communications and Information Technology, Vol. 2, pp. 1213–1216, 2005.
[11] M.B. Bell, W.C. Cathey, "The iterated Kalman filter update as a Gauss-Newton method", IEEE Transactions on Automatic Control, Vol. 38, pp. 294–297, 1993.
[12] Z.B. Yang, D.X. Zhong, F.C. Guo, Y.Y. Zhou, "Gauss-Newton iteration based algorithm for passive location by a single observer", Systems Engineering and Electronics, Vol. 29, pp. 2006–2009, 2007.
[13] L.A. Johnston, V. Krishnamurthy, "Derivation of a sawtooth iterated extended Kalman smoother via the AECM algorithm", IEEE Transactions on Signal Processing, Vol. 49, pp. 1899–1909, 2001.



Chaochen Wang received his Master's degree in Control Theory and Engineering at Nanjing University of Science and Technology, China, in 2012. He is currently an Assistant Research Fellow in School of Automation at Nanjing University of Science and Technology. His research interests include signal processing, navigation, system integration and target tracking.

Panlong Wu received his Ph.D. degree in Control Science and Engineering at Northwestern Polytechnical University, China, in 2006. He is currently an Associate Professor in School of Automation at Nanjing University of Science and Technology. His research interests include signal processing, navigation and target tracking.

Yuhao Deng received his B.S. degree in Automation at Nanjing University of Science and Technology, China, in 2014. He is currently pursuing his M.Sc. degree in Control Science and Engineering at Nanjing University of Science and Technology, China. His research interests include signal processing, navigation and target tracking.